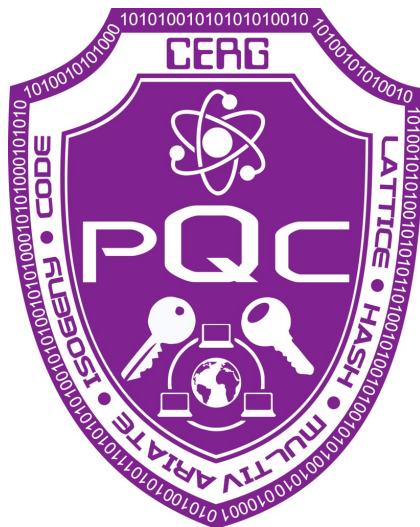


High-Speed Hardware Implementations of Post-Quantum Cryptography Digital Signature Schemes

Luke Beckwith, Robert Wallace,
Duc T. Nguyen, Kamyar Mohajerani,
and Kris Gaj



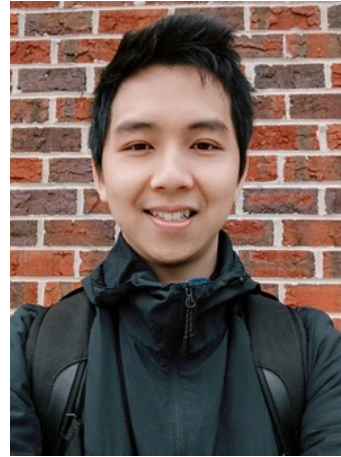
Co-Authors



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Post-Quantum Cryptography (PQC)

- Public-key cryptographic algorithms for which there are **no known attacks** using quantum computers
 - Capable of being implemented using any **traditional** methods, including **software and hardware**
 - Running efficiently on **any modern computing platforms**: PCs, tablets, smartphones, servers with FPGA accelerators, etc.
- Based entirely on traditional semiconductor VLSI technology!

The biggest revolution in cryptography since the invention of public-key cryptography in 1970s!!!

Types and Security Levels of PQC Schemes

Types

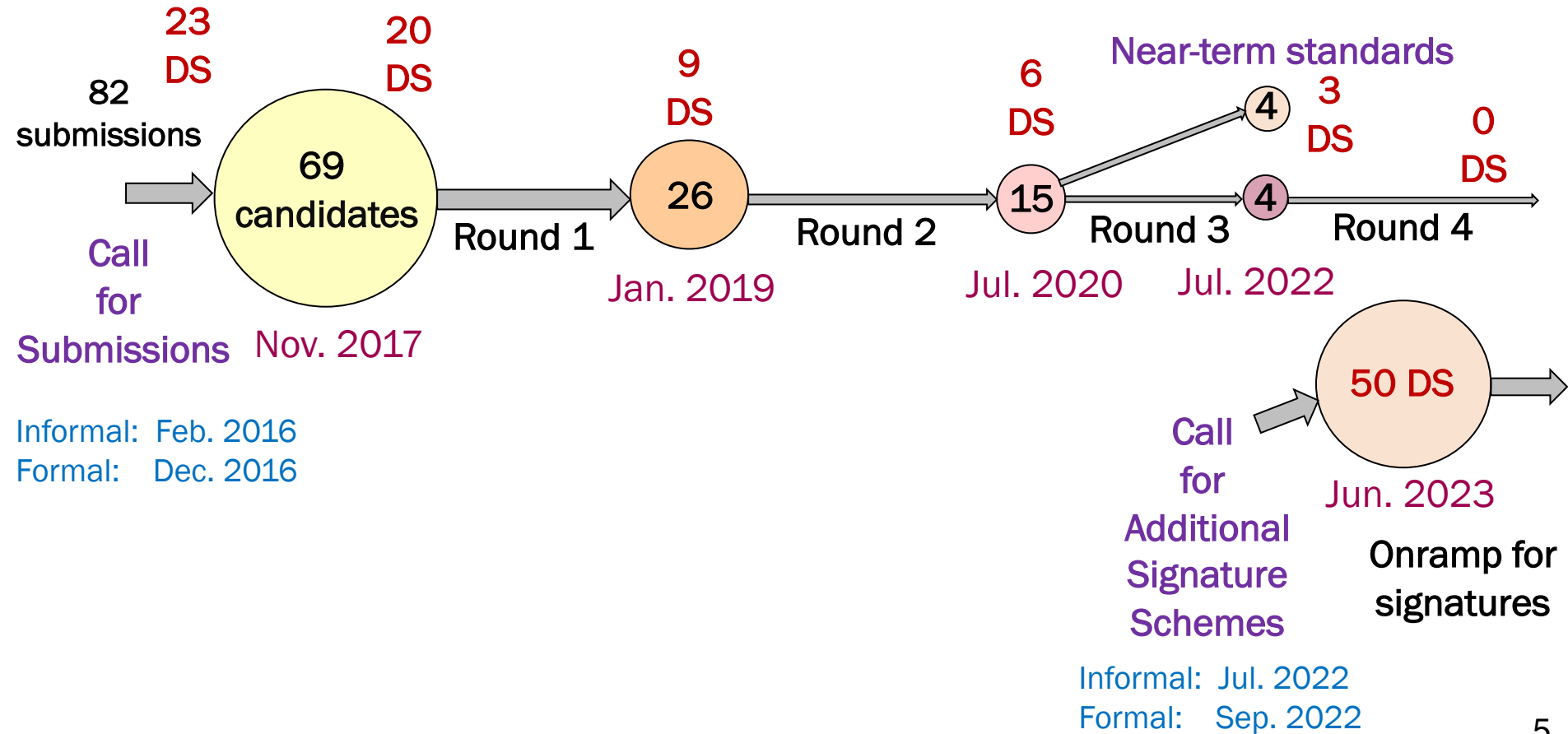
1. Public Key Encryption (PKE)
2. Key Encapsulation Mechanism (KEM)
3. Digital Signature (DS)

Security Levels

Level	Standard w/ Equivalent Security
1	AES-128
2	SHA-256
3	AES-192
4	SHA-384
5	AES-256

Only Level 5 schemes allowed in the U.S.
National Security Systems

NIST PQC Standardization Process



Focus of this Talk

Near-Term Digital Signature Standards

Lattice-based

CRYSTALS-DILITHIUM
FALCON

Symmetric-based
(hash-based)

SPHINCS+

Candidate in a New Onramp Signature Process

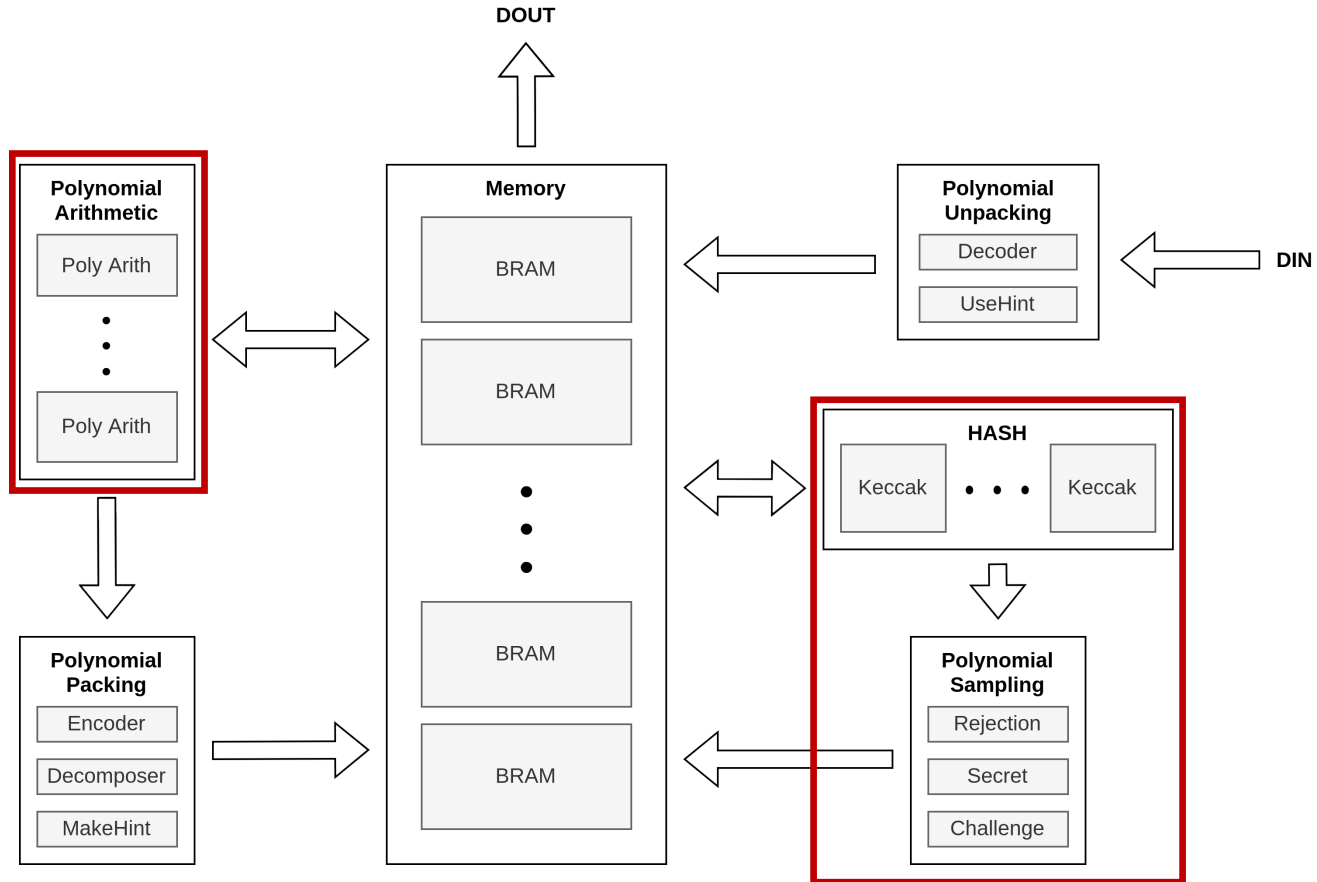
Code-based

LESS:
Linear Equivalence Signature Scheme

<

<https://www.less-project.com>

CRYSTALS-Dilithium: High-Level Architecture



Polynomial Sampling – Bottleneck no. 1

- Core elements are matrices and vectors of polynomials
- Sampled uniformly from the output of the SHA-3 extendable-output function (XOF) SHAKE

$$\begin{matrix} & & & & l \\ & & & & \underbrace{\hspace{10em}} \\ & & & & \\ k & \left\{ \begin{array}{c} \left[\begin{array}{ccc} A_{1,1} & \cdots & A_{1,l} \\ \vdots & \ddots & \vdots \\ A_{k,1} & \cdots & A_{k,l} \end{array} \right] \end{array} \right. \end{matrix}$$

$$A_{i,j} \in R_q = a_{N-1}x^{N-1} + \dots + a_1x^1 + a_0$$

where R_q is the ring $\mathbb{Z}_q[X]/(X^N + 1)$

Polynomial Sampling - Number of bytes per matrix

Parameter	Dilithium2	Dilithium3	Dilithium5
Dimensions (k, l)	(4, 4)	(6, 5)	(8, 7)
#Polynomial coefficients (N)	256		
Total number of coefficients ($k \cdot l \cdot N$)	4096	7680	14336
Coefficient Modulus (q)	$8,380,417 = 2^{23} - 2^{13} + 1$		
Total number of bytes	12 kbytes	23 kbytes	43 kbytes

Solution – Using multiple (3-4) Keccak cores in High-Speed Implementations

Polynomial Arithmetic - Dilithium Bottleneck no. 2

- Performs polynomial multiplication using the Number Theoretic Transform (NTT)

- $C(x) = A(x) \cdot B(x) = \text{NTT}^{-1}(\text{NTT}(A) \circ \text{NTT}(B)) = \text{NTT}^{-1}(\hat{A} \circ \text{NTT}(B))$,

where $\hat{A} = A$ in the NTT domain, \circ is a point-wise multiplication

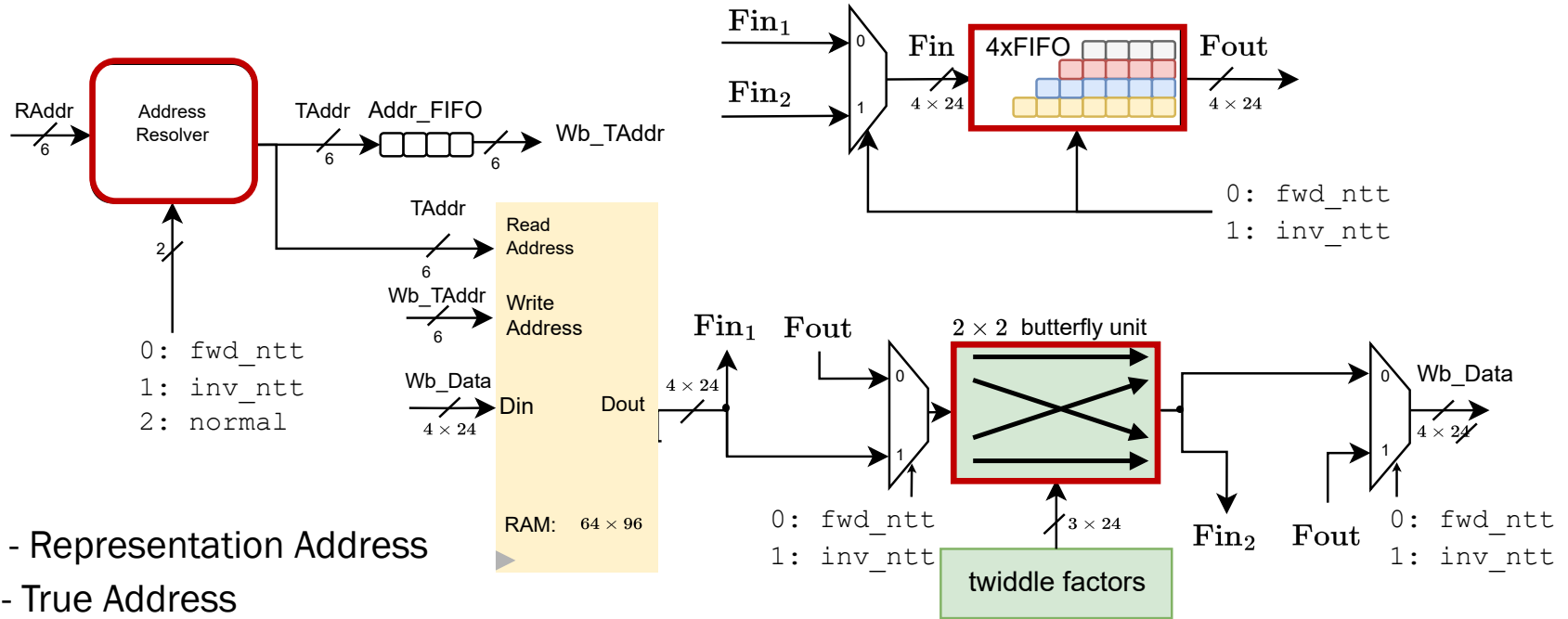
The diagram shows a vector of size k (represented by a bracket on the left) multiplied by a matrix of size $k \times l$ (represented by a bracket on top) to produce a vector of size l (represented by a bracket on the right). The matrix is labeled A and the vector is labeled s_1 . The result vector is labeled t .

Matrix A is
sampled in
the NTT-domain

- Input vector s_1 must be converted using the NTT
- Result vector t must be converted back using the Inverse NTT

l NTT transformations
 k inverse NTT transformations
 $k \cdot l$ point-wise multiplications
 $k \cdot (l-1)$ additions

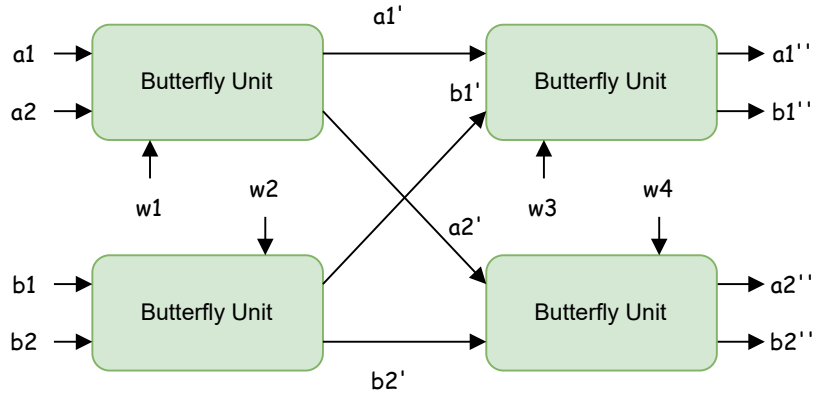
Dilithium: Polynomial Arithmetic – NTT Unit



RAddr - Representation Address
 TAddr - True Address

Number of clock cycles proportional to $O(N \log(N))$

Dilithium: NTT Unit– 2 x 2 Butterfly Unit



Operations of each Butterfly Unit

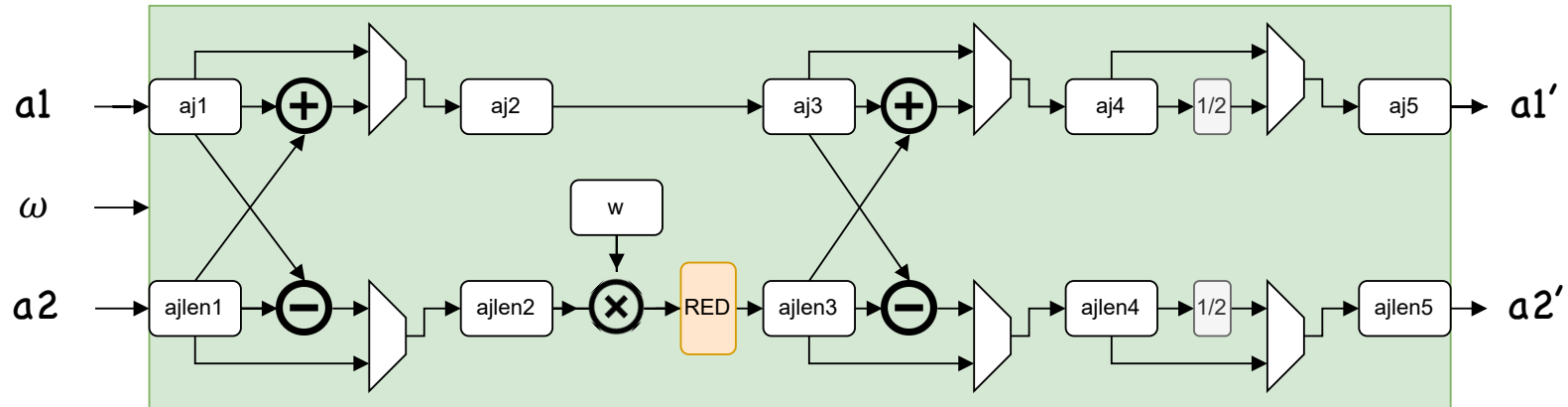
- Cooley-Tukey Butterfly (Forward NTT):

$$a1' = a1 + \omega \cdot a2 \text{ mod } q$$

$$a2' = a1 - \omega \cdot a2 \text{ mod } q$$
- Gentleman-Sande Butterfly (Inverse NTT):

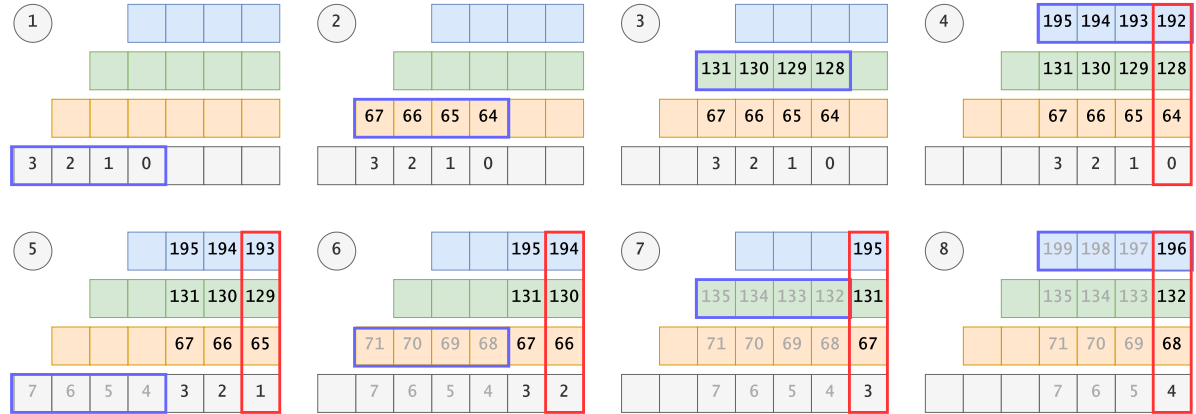
$$a1' = a1 + a2 \text{ mod } q,$$

$$a2 = (a1 - a2) \cdot \omega \text{ mod } q$$
- Inverse also requires scaling by $\frac{1}{N} = \left(\frac{1}{2}\right)^{\log_2 N}$

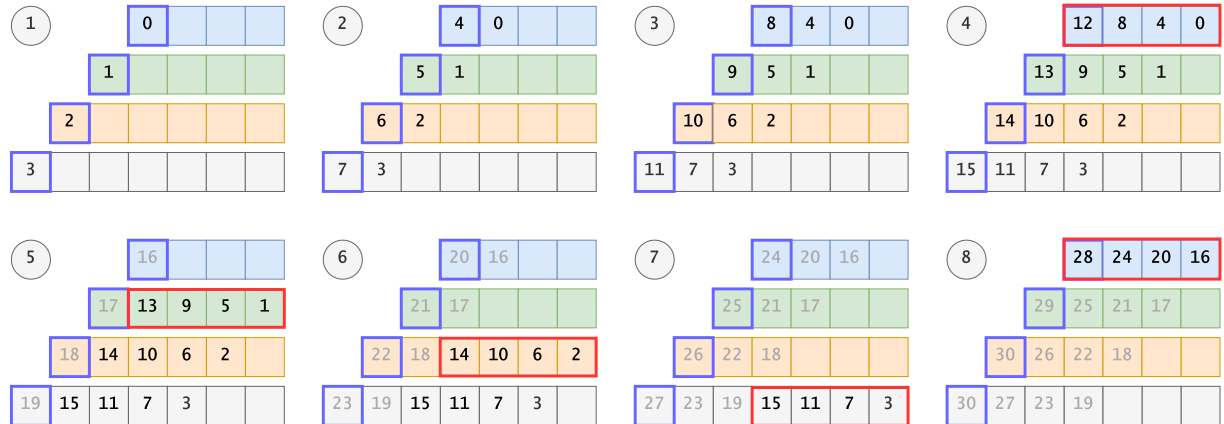


Dilithium: NTT Unit- 4xFIFO

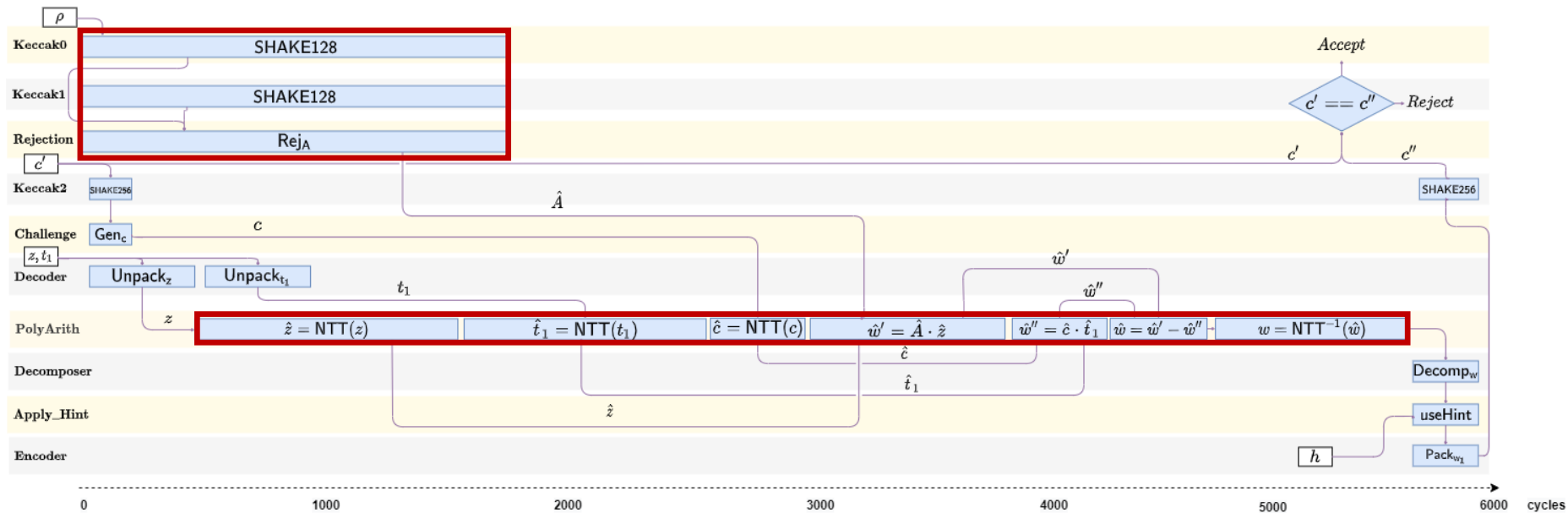
Forward NTT



Inverse NTT

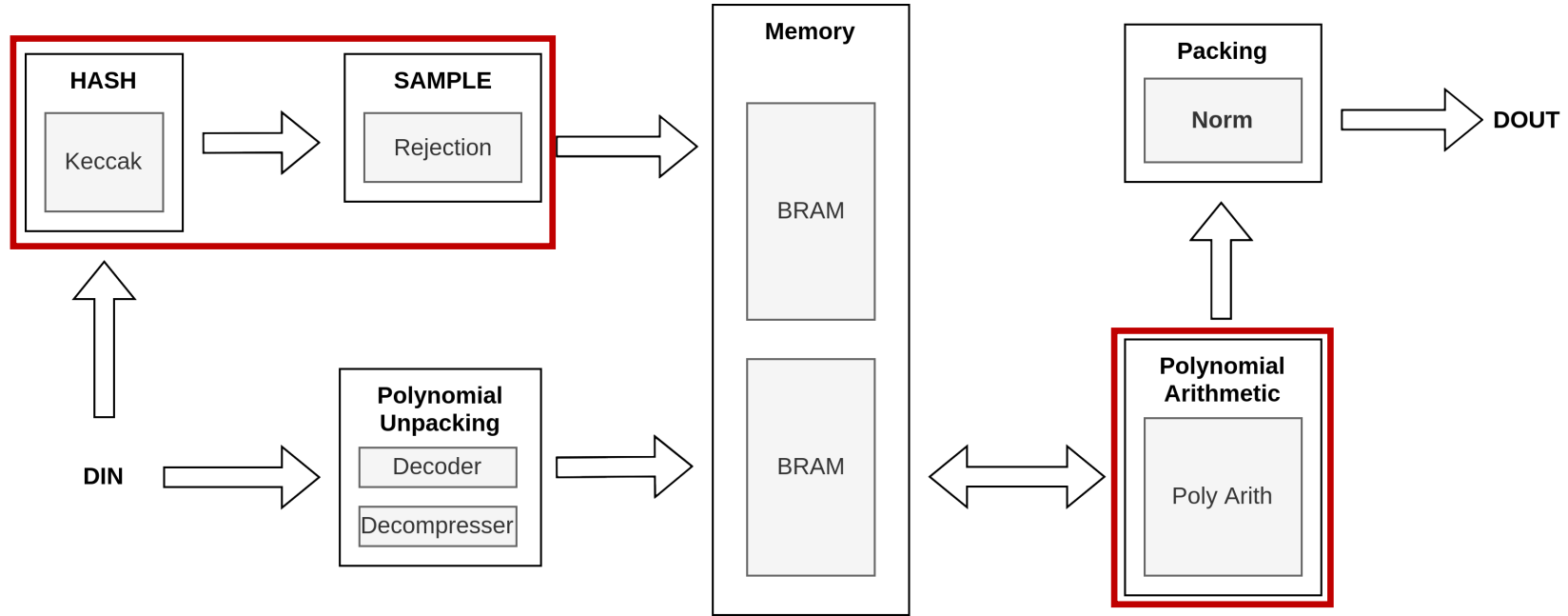


Dilithium: Example of Operation Scheduling - Verification



Bottlenecks: Sampling of A matrix + calculation of $Az - ct_1$

FALCON (Verification only): High-Level Architecture



- KeyGen and Sign are much more complicated
- Require implementing floating point Fast Fourier Transform (FFT)
- Hardware implementations not reported yet

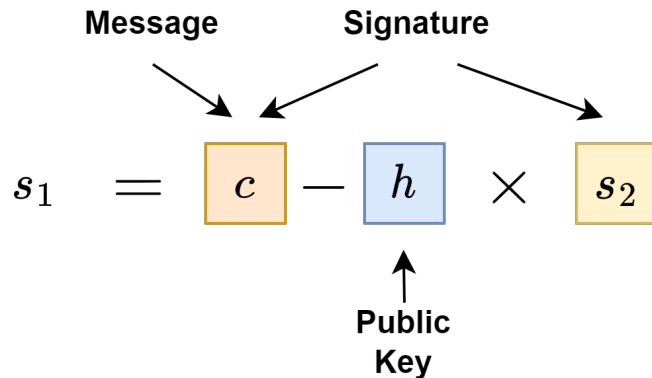
FALCON: Major Operations of Verification

Given message M , signature ($r \in \{0,1\}^{320}, s_2 \in Z_q[x]$), and the public key ($h \in Z_q[x]$), calculate:

$$c \leftarrow \text{HashToPoint}(r || M)$$

$$s_1 = c - h * s_2$$

If $\|s_1, s_2\|^2 = \sum_{i=0}^{n-1} (s_1^2[i] + s_2^2[i])$ is less than the parameter value β , accept



Parameter	FALCON512	FALCON1024
Coefficient Modulus (q)	12,289 = $12 \cdot 2^{10} + 1$	
Number of Polynomial Coefficients (N)	512	1024

Latency of Major Operations in μs : Level 5 – Artix-7 FPGA

Lightweight

1 x NTT, 1 Keccak

High-Speed

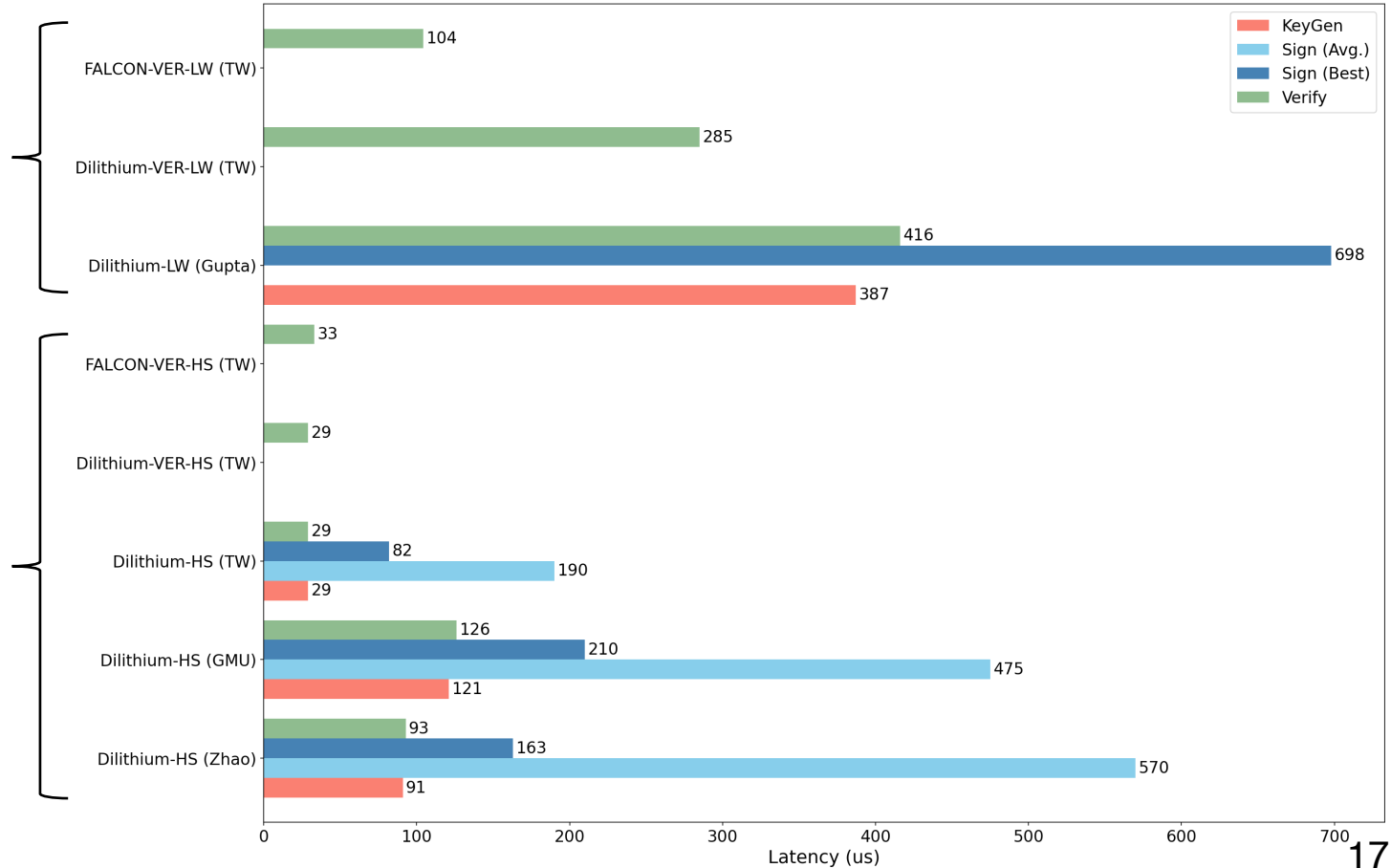
1 x NTT, 1 Keccak

4 x NTT, 4 Keccak

4 x NTT, 4 Keccak

2 x NTT, 3 Keccak

1 x NTT, 1 Keccak



Resource Utilization: Level 5 – Artix-7 FPGA

Lightweight

1 x NTT, 1 Keccak

High-Speed

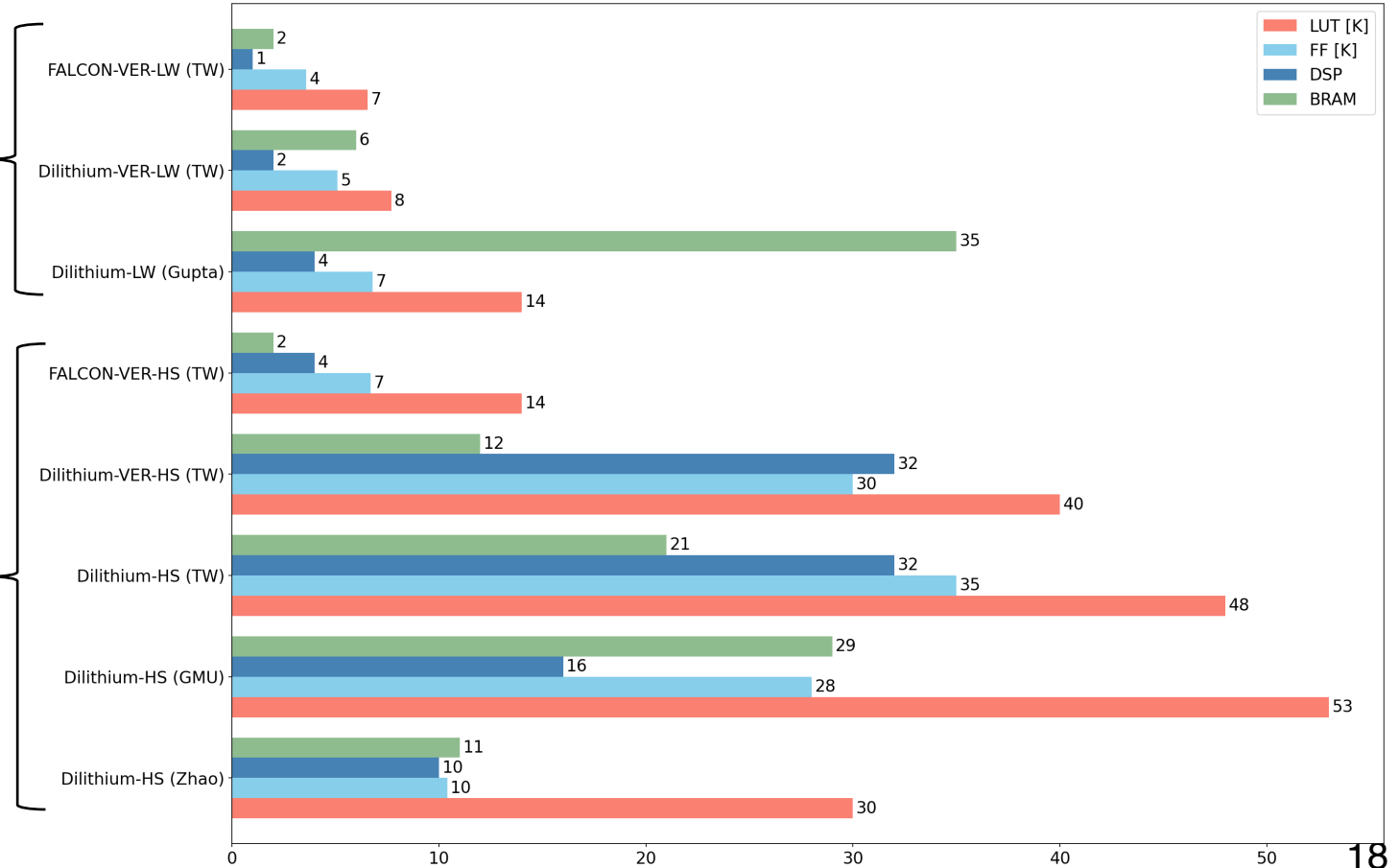
1 x NTT, 1 Keccak

4 x NTT, 4 Keccak

4 x NTT, 4 Keccak

2 x NTT, 3 Keccak

1 x NTT, 1 Keccak



LESS: Conversion of a Matrix to the Reduced Row Echelon Form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 6 & 0 & 4 \\ 0 & 1 & 5 & 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 1 & 6 & 3 & 4 \end{bmatrix}$$

Reduced Row Echelon Form (RREF):

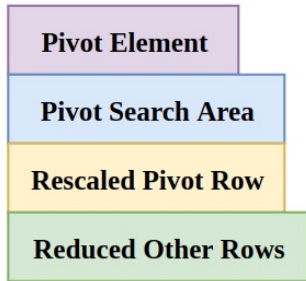
- (1) the leading entry of every nonzero row is to the right of the leading entry of every row above it
- (2) the leading entry in every non-zero row is 1,
- (3) each column containing a leading 1 has zeros in all of its other entries.

LESS: Conversion to RREF – Example – Part 1

$n=7$

$k=3$

$q=7$



A.
$$\begin{bmatrix} 2 & 2 & 3 & 3 & 1 & 4 & 3 \\ 3 & 3 & 1 & 5 & 1 & 4 & 3 \\ 5 & 3 & 1 & 2 & 2 & 2 & 6 \end{bmatrix}$$

B.
$$\begin{bmatrix} 2 & 2 & 3 & 3 & 1 & 4 & 3 \\ 3 & 3 & 1 & 5 & 1 & 4 & 3 \\ 5 & 3 & 1 & 2 & 2 & 2 & 6 \end{bmatrix}$$

C.
$$\begin{bmatrix} 2 & 2 & 3 & 3 & 1 & 4 & 3 \\ 3 & 3 & 1 & 5 & 1 & 4 & 3 \\ 5 & 3 & 1 & 2 & 2 & 2 & 6 \end{bmatrix}$$

D.
$$\begin{bmatrix} 1 & 1 & 5 & 5 & 4 & 2 & 5 \\ 3 & 3 & 1 & 5 & 1 & 4 & 3 \\ 5 & 3 & 1 & 2 & 2 & 2 & 6 \end{bmatrix}$$

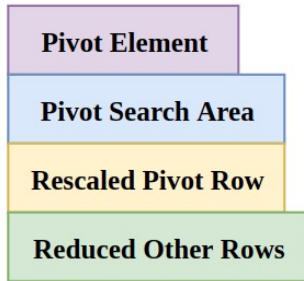
E.
$$\begin{bmatrix} 1 & 1 & 5 & 5 & 4 & 2 & 5 \\ 0 & 0 & 0 & 4 & 3 & 5 & 2 \\ 0 & 5 & 4 & 5 & 3 & 6 & 2 \end{bmatrix}$$

LESS: Conversion to RREF – Example – Part 2

$n=7$

$k=3$

$q=7$



F.
$$\begin{bmatrix} 1 & 1 & 5 & 5 & 4 & 2 & 5 \\ 0 & 0 & 0 & 4 & 3 & 5 & 2 \\ 0 & 5 & 4 & 5 & 3 & 6 & 2 \end{bmatrix}$$

G.
$$\begin{bmatrix} 1 & 1 & 5 & 5 & 4 & 2 & 5 \\ 0 & 0 & 0 & 4 & 3 & 5 & 2 \\ 0 & 5 & 4 & 5 & 3 & 6 & 2 \end{bmatrix}$$

Diagram G shows a purple box around the element 5 in the second column of the third row. A curved arrow points from this element to the second column of the second row, and another curved arrow points from the second column of the second row back to the element 5 in the second column of the third row.

H.
$$\begin{bmatrix} 1 & 1 & 5 & 5 & 4 & 2 & 5 \\ 0 & 1 & 5 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 4 & 3 & 5 & 2 \end{bmatrix}$$

I.
$$\begin{bmatrix} 1 & 0 & 0 & 4 & 2 & 5 & 6 \\ 0 & 1 & 5 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 4 & 3 & 5 & 2 \end{bmatrix}$$

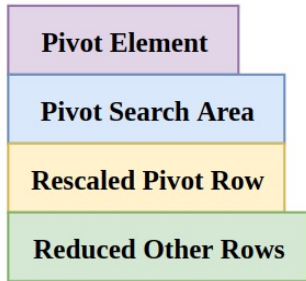
Diagram H shows a purple box around the element 1 in the second column of the second row, and a yellow box highlighting the entire second row. Diagram I shows a green box highlighting the first and third rows, a purple box around the element 1 in the second column of the second row, and a blue box highlighting the second, third, and fourth columns of the third row.

LESS: Conversion to RREF – Example – Part 2

$n=7$

$k=3$

$q=7$



$$\text{J. } \begin{bmatrix} 1 & 0 & 0 & 4 & 2 & 5 & 6 \\ 0 & 1 & 5 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 4 & 3 & 5 & 2 \end{bmatrix}$$

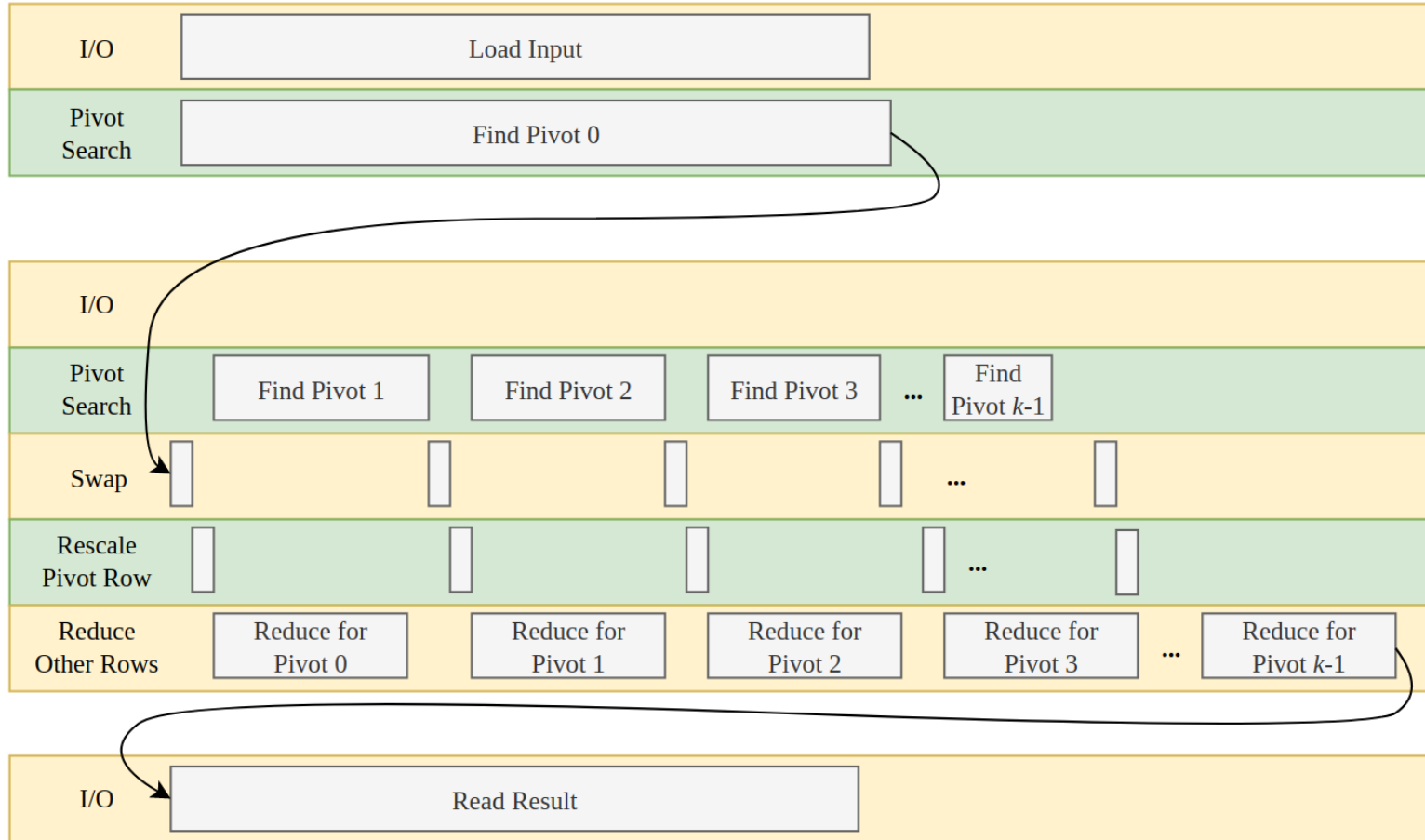
$$\text{K. } \begin{bmatrix} 1 & 0 & 0 & 4 & 2 & 5 & 6 \\ 0 & 1 & 5 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 4 & 3 & 5 & 2 \end{bmatrix}$$

$$\text{L. } \begin{bmatrix} 1 & 0 & 0 & 4 & 2 & 5 & 6 \\ 0 & 1 & 5 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 6 & 3 & 4 \end{bmatrix}$$

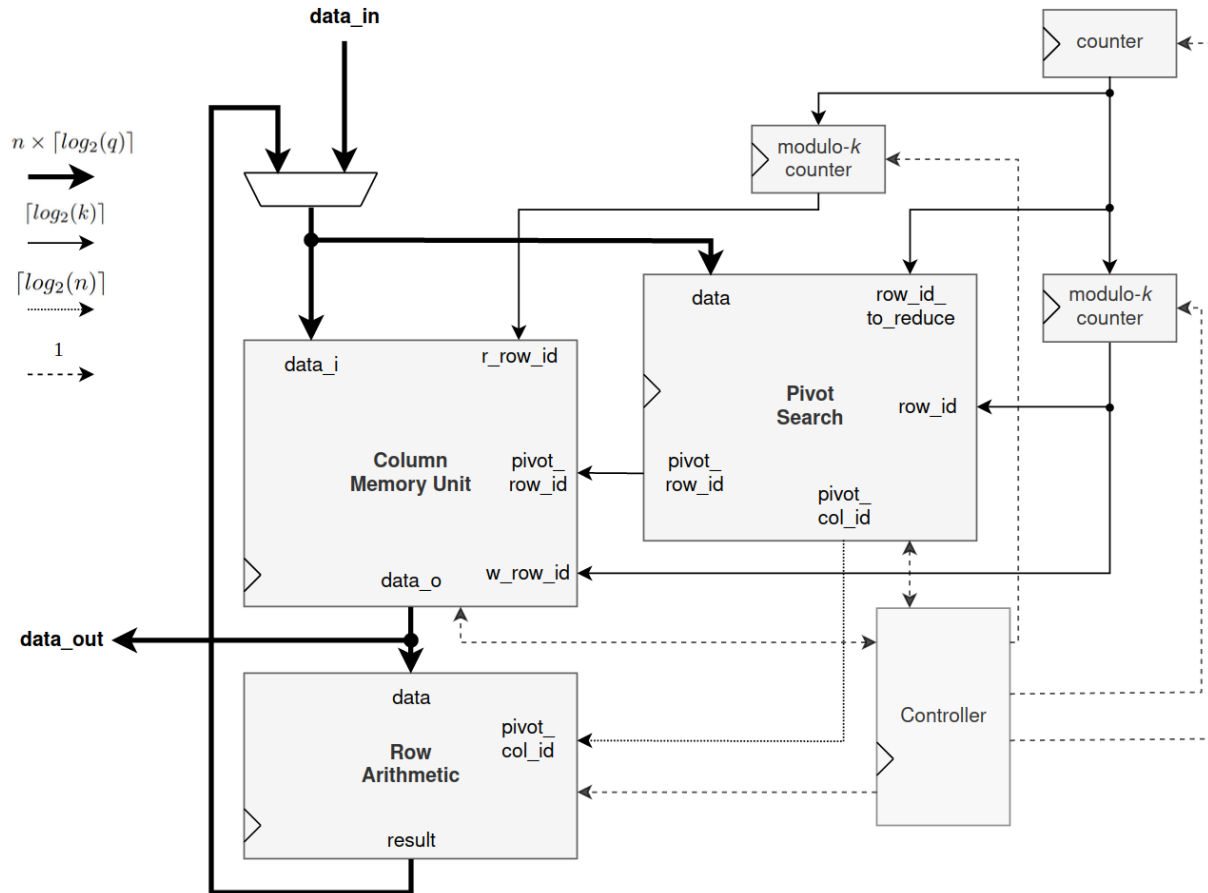
$$\text{M. } \begin{bmatrix} 1 & 0 & 0 & 0 & 6 & 0 & 4 \\ 0 & 1 & 5 & 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 1 & 6 & 3 & 4 \end{bmatrix}$$

$$\text{N. } \begin{bmatrix} 1 & 0 & 0 & 0 & 6 & 0 & 4 \\ 0 & 1 & 5 & 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 1 & 6 & 3 & 4 \end{bmatrix}$$

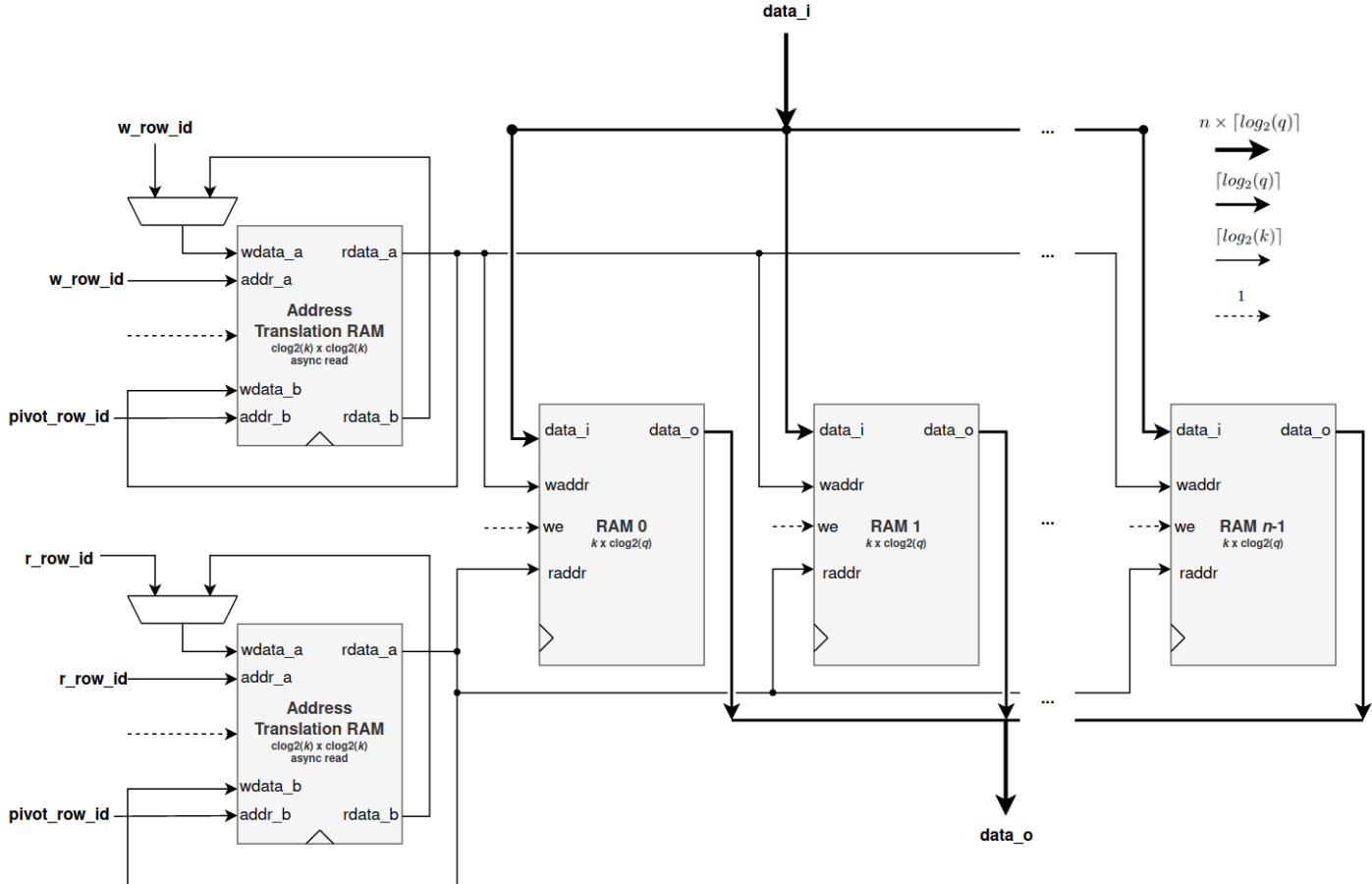
Scheduling of Major Operations within RREF



LESS: RREF Unit: Top-Level Block Diagram



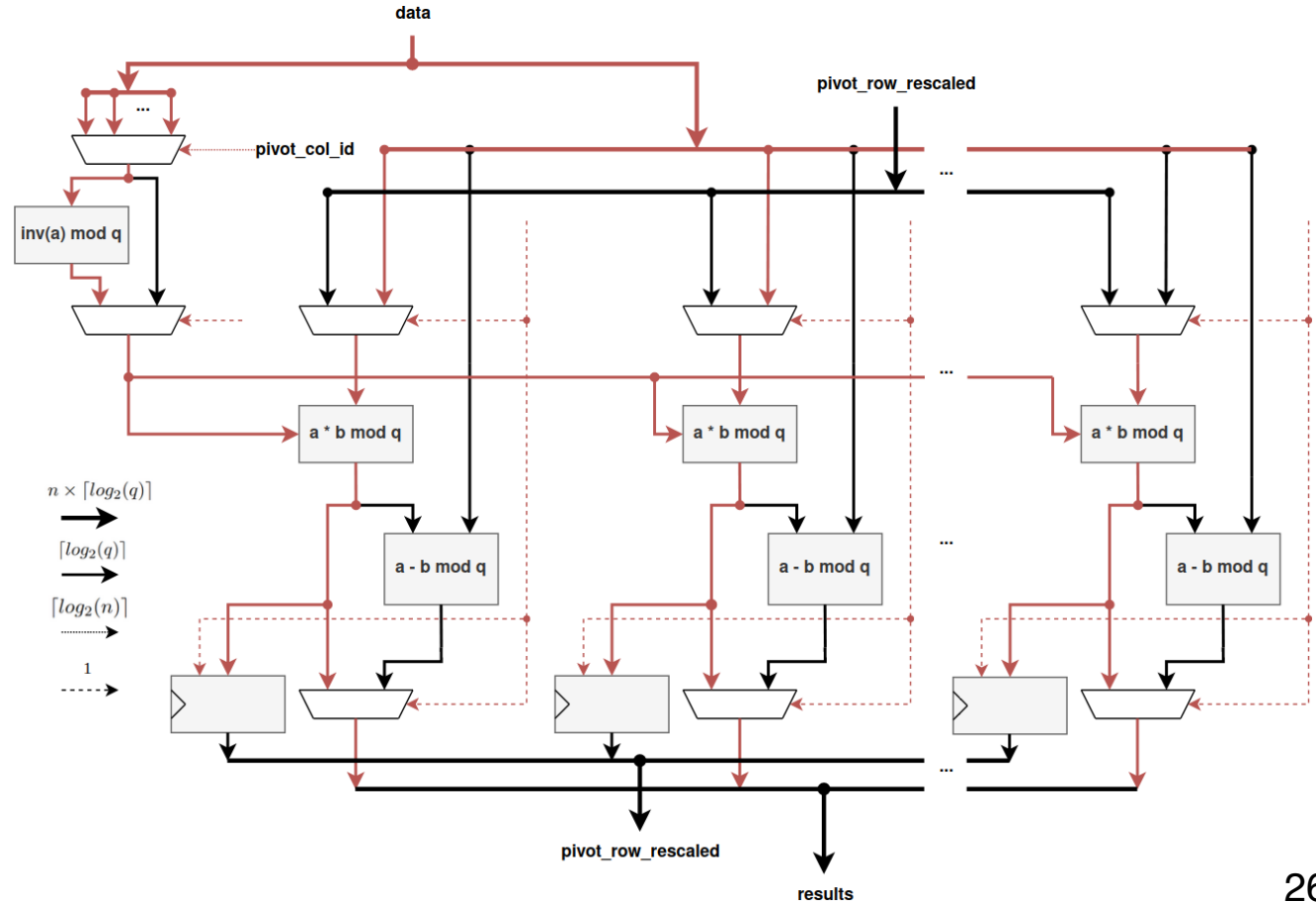
LESS: RREF Unit: Column Memory Unit



LESS: RREF Unit: Row Arithmetic Unit – Rescale Pivot Row

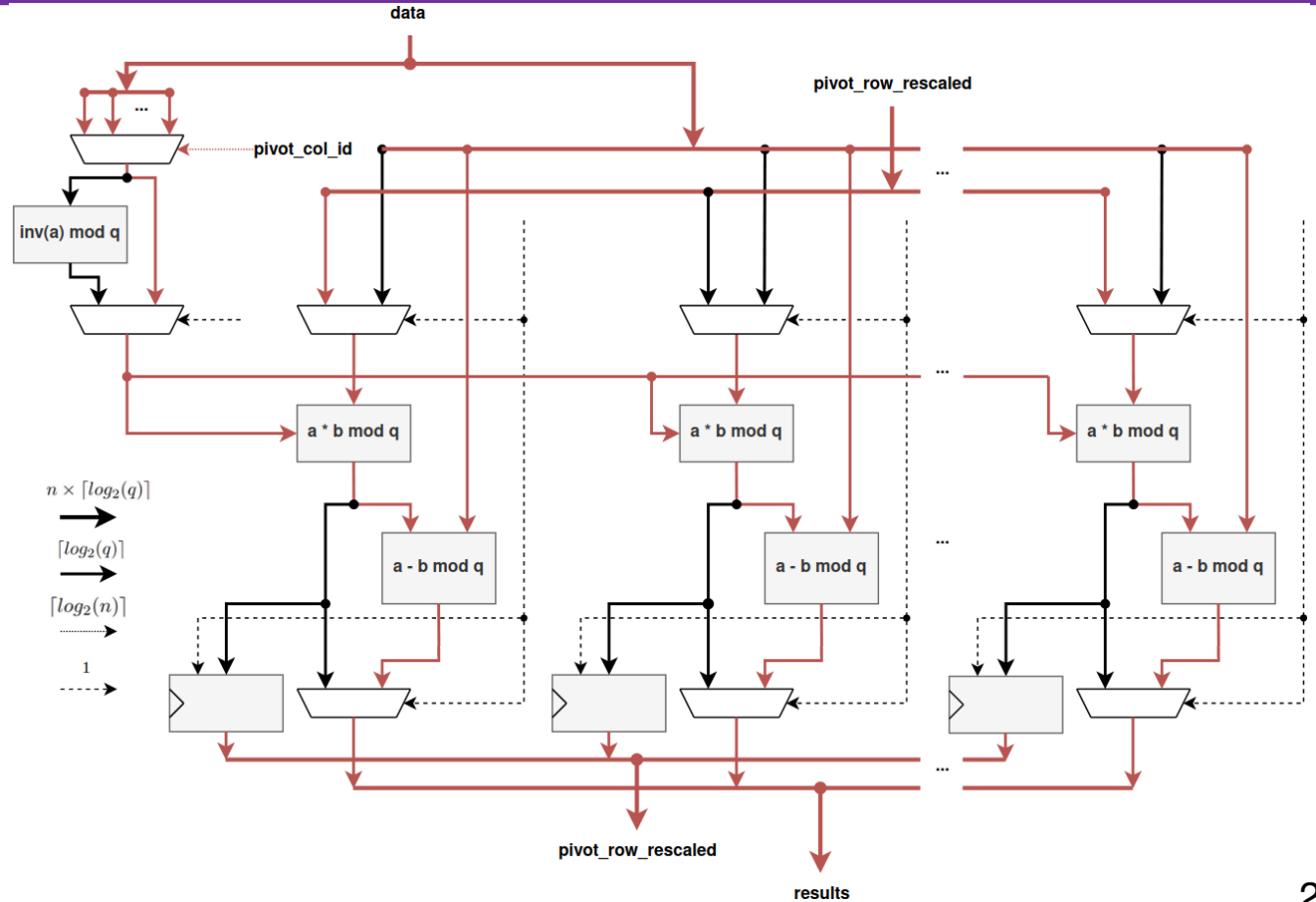
2	2	3	3	1	4	3
3	3	1	5	1	4	3
5	3	1	2	2	2	6

1	1	5	5	4	2	5
3	3	1	5	1	4	3
5	3	1	2	2	2	6

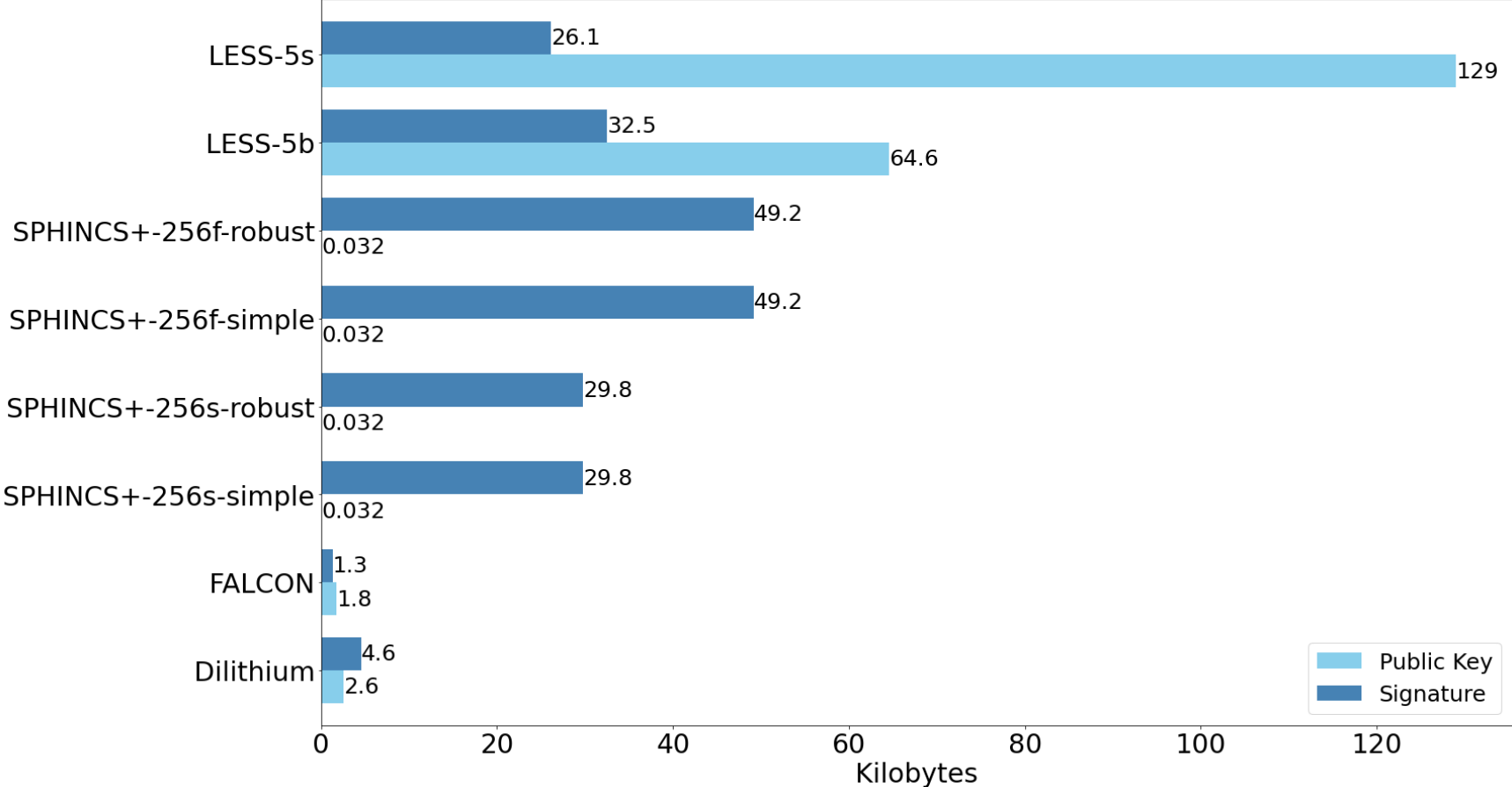


LESS: RREF Unit: Row Arithmetic Unit – Reduce Other Rows

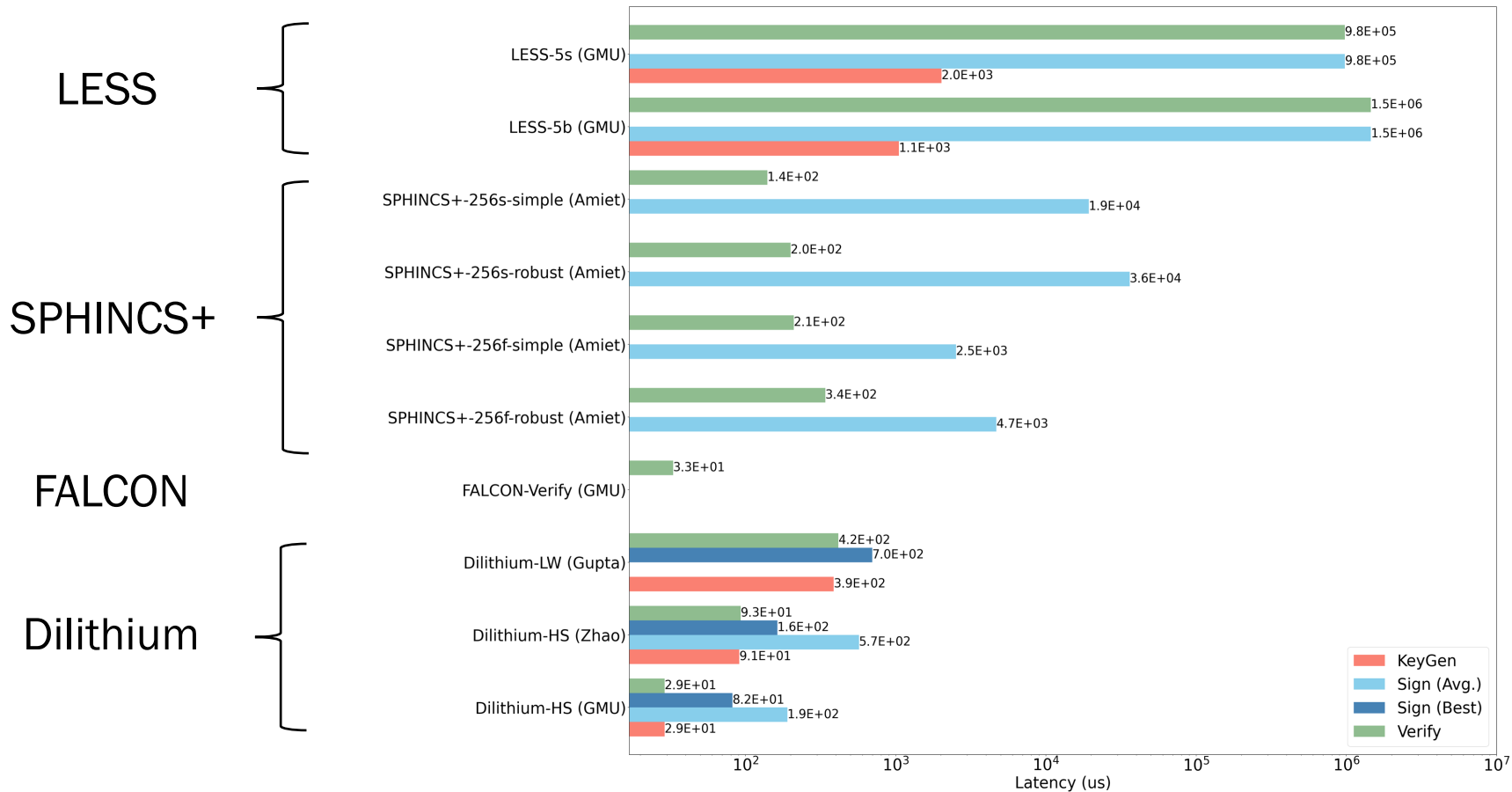
1	1	5	5	4	2	5
3	3	1	5	1	4	3
5	3	1	2	2	2	6
1	1	5	5	4	2	5
0	0	0	4	3	5	2
0	5	4	5	3	6	2



Public Key and Signature Sizes: LESS vs. Near-Term Standards



Latencies in μs on Artix-7: LESS vs. Near-Term Standards



Conclusions

- FALCON

- + Smallest sum signature size + public key size (certificate size)
- + Very efficient verification
- Complex key generation and signing

- Dilithium

- + Second smallest sum signature size + public key size (certificate size)
- + Efficient key generation, signing, and verifying

- LESS

- + Security dependent on a different problem than all other near-term NIST standards
- +/- Signature sizes comparable to SPHINCS+ but larger than in FALCON & Dilithium
- Longer public-key sizes than in all other near-term NIST standards
- Orders of magnitude longer execution times for signing and verifying

Q&A

Thank You!

Questions?



Comments?

CERG: <http://cryptography.gmu.edu>

ATHENa: <http://cryptography.gmu.edu/athena>

Menu Field: PQC